Equations différentielles et primitives : corrigé de l'évaluation formative.

1. a) $\int (2x^3 - x^2 + 5)\,dx = \frac{x^4}{2} - \frac{x^3}{3} + 5x + c.$

b) $\int \frac{x^3+1}{x}\,dx = \int \left(x^2 + \frac{1}{x}\right) dx = \frac{x^3}{3} + \ln|x| + c$

c) $\int \frac{x^2}{(x^3-7)^3}\,dx = \frac{1}{3}\int 3x^2 \cdot (x^3-7)^{-3}\,dx = \frac{-1}{6(x^3-7)^2} + c$

d) $\int \frac{x^2}{x^3-7}\,dx = \frac{1}{3}\cdot \ln|x^3-7| + c$

e) $\int e^{-4x}\,dx = -\frac{1}{4}e^{-4x} + c$

f) $\int x^{10}\cdot \ln x\,dx = \frac{x^{11}}{11}\cdot \ln x - \int \frac{1}{x}\cdot\frac{x^{11}}{11}\,dx$

$u = \ln x \rightarrow u' = \frac{1}{x}$

$v' = x^{10} \rightarrow v = \frac{x^{11}}{11}$

$= \frac{x^{11}}{11}\cdot \ln x - \frac{1}{11}\int x^{10}\,dx$

$= \frac{x^{11}}{11}\cdot\left(\ln x - \frac{1}{11}\right) + c.$

g) $\int \frac{1}{\sqrt{1-25x^2}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,\frac{du}{5} = \frac{1}{5}\arcsin(5x) + c$

$u = 5x$
$du = 5dx$

h) $\int \frac{1}{e^x+1}\,dx = \int \frac{1}{\frac{1}{t}+1}\cdot\frac{-1}{t}\,dt = \int \frac{-1}{\frac{1+t}{\cancel{t}}}\cdot\frac{1}{\cancel{t}}\,dt$

$x = -\ln t$

$dx = -\frac{1}{t}\,dt$

$e^x = (e^{\ln t})^{-1} = \frac{1}{t}$

$= -\ln|1+t| + c$

$= -\ln|1+e^{-x}| + c$

i) $\int \frac{2x+1}{x^2-4}\,dx = \int \frac{5/4}{x-2}\,dx + \int \frac{3/4}{x+2}\,dx$

$\frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2) + B(x-2)}{x^2-4}$

si $x = 2 \rightarrow 4A = 5$

si $x = -2 \rightarrow -4B = -3$

$= \frac{5}{4}\ln|x-2|$

$+ \frac{3}{4}\ln|x+2| + c$

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2. a) $v(t) = -8t + 42$

b) $d(t) = -4t^2 + 42t$

c) $v(t) = 0 \Leftrightarrow t = 5{,}25\ (s)$

$d_{\text{arrêt}} = d(5{,}25) = 110{,}25\ (m).$