

Solutions des exercices de préparation au test de trigonométrie.

$$1. \quad \cos \frac{7\pi}{12} = \cos \left(\frac{\pi}{3} + \frac{\pi}{4} \right) = \cos \frac{\pi}{3} \cdot \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \cdot \sin \frac{\pi}{4}$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\cot \frac{7\pi}{12} = \frac{1}{\tan \left(\frac{\pi}{3} + \frac{\pi}{4} \right)} = \frac{1 - \tan \frac{\pi}{3} \cdot \tan \frac{\pi}{4}}{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}} = \frac{1 - \sqrt{3}}{\sqrt{3} + 1}$$

$$2. \quad \cos \left(\frac{\pi}{6} - x \right) - \sin \left(\frac{\pi}{3} + x \right)$$

$$= \cos \frac{\pi}{6} \cdot \cos x + \sin \frac{\pi}{6} \cdot \sin x - \sin \frac{\pi}{3} \cdot \cos x - \sin x \cdot \cos \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2} \cdot \cos x + \frac{1}{2} \cdot \sin x - \frac{\sqrt{3}}{2} \cdot \cos x - \frac{1}{2} \cdot \sin x = 0.$$

Autre façon :

$$\cos \left(\frac{\pi}{6} - x \right) - \cos \left[\frac{\pi}{2} - \left(\frac{\pi}{3} + x \right) \right]$$

$$= \cos \left(\frac{\pi}{6} - x \right) - \cos \left(\frac{\pi}{6} - x \right) = 0.$$

$$3. \quad a) \quad \cos 2a = 2 \cdot \cos^2 a - 1 = 2 \cdot \frac{16}{25} - 1 = \frac{7}{25}.$$

$$b) \quad \sin 2a = 2 \cdot \sin a \cdot \cos a$$

$$= 2 \cdot \sqrt{1 - \cos^2 a} \cdot \cos a \quad (\text{car } a \in [0, \frac{\pi}{2}])$$

$$= 2 \cdot \sqrt{1 - \frac{16}{25}} \cdot \frac{4}{5} = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}.$$

$$c) \quad \sin(a+b) = \sin a \cdot \cos b + \sin b \cdot \cos a$$

$$\sin a = \frac{3}{5} \quad (\text{voisin } (b)) \quad \text{et} \quad \cos b = -\sqrt{1 - \sin^2 b}$$

$$(\text{car } b \in [\frac{\pi}{2}, \pi])$$

$$= -\sqrt{1 - \frac{1}{9}} = -\frac{2\sqrt{2}}{3}.$$

$$\sin(a+b) = \frac{3}{5} \cdot \frac{-2\sqrt{2}}{3} + \frac{1}{3} \cdot \frac{4}{5} = \frac{-6\sqrt{2} + 4}{15}$$

$$= \frac{-2\sqrt{2}}{5} + \frac{4}{15}.$$

$$d) \quad \cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$$

$$= \frac{4}{5} \cdot \frac{-2\sqrt{2}}{3} + \frac{3}{5} \cdot \frac{1}{3} = \frac{-8\sqrt{2}}{15} + \frac{1}{5}.$$

$$e) \quad \tan b = \frac{\sin b}{\cos b} = \frac{\frac{1}{3}}{\frac{-2\sqrt{2}}{3}} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4} \rightarrow \dots$$